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ABSTRACT

Analysts differ quite heatedly over the use of principal components rather than principle axis factor analysis. The difference between the two approaches involves the entries used on the diagonal of the matrix of associations that is analyzed. This paper uses an actual data set (N=539; variables =98) to illustrate that these two methods converge in their results when the number of variables is large, even if score reliabilities are low. (Contains 2 tables and 13 references.) (Author/SLD)

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Principal Components versus Principle Axis Factors:

When Will We Ever Learn?

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ED 449 215

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Abstract

Analysts differ quite heatedly over the use of principal components as against principle axis factor analysis. The present paper uses an actual data set (n = 539; variables = 98) to illustrate that these two methods converge in their results when the number of variables is large, even if score reliabilities are low.

As Thompson and Daniel (1996) noted,

> Analysts differ quite heatedly over the utility of principal components as compared to common or principle [axis] factor analysis [i.e., PAF].... The differences between the two approaches involves the entries used on the diagonal of the matrix of associations that is analyzed. When a correlation matrix is analyzed, principal components analysis uses ones on the diagonal whereas common factor analysis uses estimates of reliability, usually estimated through an iterative process. (p. 201)

These differences are so heated that an entire special issue (Mulaik, 1992) on this controversy was published in *Multivariate Behavioral Research*.

Thompson (1992) argued that the practical difference between the methods is often negligible in terms of interpretation. Differences in results will decrease as (a) the measured variables have greater score reliability or (b) the number of variables measured increases (Gorsuch, 1986).

Regarding the first influence, the higher the score reliability is for a variable, the closer the PAF entry on the diagonal is to one, which is what is used by PCA. So in cases where the reliability of variable scores is high, PCA AND PAF will converge in their estimates. Of course, understanding this influence requires understanding of basic concepts of reliability; useful treatments of these concepts are provided by Dawson (1999),

Reinhardt (1996), Thompson and Vacha-Haase (2000), and Vacha-Haase, Kogan and Thompson (2000).

Regarding the second influence, as the number of variables increases, so does the total number of entries on the matrix of associations. The influence of the diagonal entries then has less influence on the solution, because the proportion of entries on the diagonal decreases exponentially as more variables are measured (cf. Snook & Gorsuch, 1989). For examples, with 10 measured variables there are 10 diagonal entries out of 100 total entries (i.e., 10.0%), but with 30 measured variables there are 30 diagonal entries out of 900 total entries (i.e., 3.3%), and with 50 measured variables there are 50 diagonal entries out of 2500 total entries (i.e., 2.0%). So even when the scores on the variables are not very reliable, PCA and PAF will still tend to converge if the number of variables is large.

The present paper provides a heuristic exploration of these influences. Data from 539 college students on the *Career Assessment Diagnostic Inventory* (CADI) were the basis for the comparison. The CADI is a 98-item multidimensional measure of career-indecision factors (Vidal-Brown & Thompson, in press).

The data provide a useful comparison of PCA versus PAF results, because the rank of the matrix is large. In this case, the diagonal represents roughly 1% of the entries in the correlation matrix (i.e., 98 / [96 x 98] = 98 / 9604 = 1.02%).

Results

Table 1 presents the varimax-rotated pattern/structure

coefficients from a *principal components* analysis of the data. Table 2 presents the varimax-rotated pattern/structure coefficients from a *principle axis factor analysis* of the data.

INSERT TABLES 1 AND 2 ABOUT HERE

Summary

The present results illustrate a dynamic that we as a field seem to continue to relearn: PCA and PAF structures tend to be comparable when the number of factored entities is large. This true regardless of whether we are factoring variables, people, or time (Thompson, 2000). Of course, we rarely apply factor analytic methods unless there are a fair number of variables (or people, when we factor people), because the structure of few variables is usually uninteresting. In these cases results across the two methods would generally be *expected* to converge. Research analytic practice as regards factor analysis would improve if more researchers understood these and related dynamics (Henson & Roberts, in press).

The order of the present factors does differ somewhat across the analyses (i.e., factors II and III are transposed in their orders). However, in general the order in which a factor is extracted across methods or samples is usually not of any particular interest, unless there is a theory present that predicts a particular factor order. This is not the usual case.

A comparison of the factors reported in Tables 1 and 2 shows that the magnitudes of the absolute values of the pattern/structure

coefficients across the two analyses are slightly smaller for the principle axis factor results. This pattern is typical. However, the critical thing to note is that, notwithstanding this dynamic, *the interpretations of the factors across the two analyses would be comparable*. That is were the rubber meets the road in the factor analysis endeavor!

These results illustrate that principal components is a perfectly reasonable analytic choice in many research situations. And there is a unique feature of principal components that may make this choice particularly appealing. In many cases researchers use the pattern coefficients to compute factor scores, which are then used in subsequent analyses (e.g., MANOVA, descriptive discriminant analysis). The correlations of the factors with each other will only exactly match the correlations of the factor scores with each other in a principal components analysis (Wells, 1999).

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Table 1
Varimax-rotated Principal Components
Pattern/Structure Coefficients

	Component					
Item	I	II	III	IV	V	VI
Q7	.07887	.08909	.02415	.63562	.03689	.09586
Q13	.13709	.05133	.06022	.69143	-.18760	.00782
Q19	.13815	.01652	.05506	.66465	.05833	-.15052
Q25	.12154	.05949	.03007	.66873	.20616	-.11626
Q37	.31287	.14320	.06437	.46758	.10381	-.09002
Q43	.11905	.01245	.07926	.77720	.02738	-.05259
Q49	.15164	.04622	.02527	.68634	-.00668	-.02000
Q55	.12632	.01154	.00141	.62046	.00935	-.15481
Q61	.09232	.09182	.02725	.70489	-.03675	.02068
Q67	.05714	.22961	-.00655	.61141	.02291	-.15518
Q73	.02064	.00687	-.01387	.51325	.04453	-.12692
Q79	.10110	.04632	.02687	.64377	-.03163	.01930
Q85	.09887	.12241	-.00223	.66185	.02455	-.06492
Q31	-.16366	-.05643	-.04305	-.61090	.13596	.12373
Q23	-.12359	-.08476	-.03227	-.12605	.42527	.06964
Q29	-.02426	.04688	.13122	.18306	.31617	.08176
Q35	.06790	.11468	.04622	-.04877	.59533	-.01653
Q41	.20313	.13231	.13851	.16007	.42829	.01519
Q47	.07428	.02833	-.01084	-.17398	.61032	-.08667
Q71	.11512	.08913	.03513	.13555	.67717	.03701
Q83	.07542	.05171	.03442	.05129	.72948	.00241
Q88	.07373	.17803	.07899	.32821	.58570	-.16080
Q91	.25491	.39270	.31058	.16107	.39127	.06281
Q94	-.16583	-.11050	-.02469	-.17395	.58422	.13671
Q96	.34568	.28542	.08962	.20987	.27124	-.02696
Q98	-.09791	-.01806	-.05575	-.28087	.44611	.01340
Q100	.12876	.06726	.07499	.09948	.70104	.04682
Q102	.08636	.18442	.07209	.14082	.71180	.02109
Q104	.14362	-.02389	.07302	.07097	.46348	-.07328
Q59	.18021	-.13390	-.13203	.15810	-.46148	-.02732
Q65	-.05622	-.11287	-.06921	.05008	-.45110	-.00918
Q77	.03998	-.11052	.01541	-.12877	-.56432	.12491
Q2	.16258	.71759	.21406	.08713	-.03753	-.07993
Q8	.10437	.63515	.17359	.12309	.08483	-.09590
Q14	.20330	.73498	.27911	.10062	.00312	-.06818
Q20	.19366	.65359	.11139	.18735	.05922	-.05972
Q26	.16520	.76457	.22325	.06582	.11998	-.03925
Q32	.05863	.53531	.26602	.06769	.10418	-.04541
Q38	.20485	.75350	.21392	.12144	.05996	-.03402
Q62	-.21427	-.63565	-.20654	.12264	-.10739	.08417
Q68	.14502	.68034	.14467	.09192	.10500	.01250
Q74	.22322	.69382	.24065	.12092	.16671	-.01681
Q80	.19517	.68807	.27268	.09175	.14788	-.02113
Q44	-.11976	-.66466	-.21615	-.01648	-.04788	.06915
Q50	-.24263	-.62987	-.07986	-.07596	-.07752	.11871
Q56	-.18530	-.42099	-.07515	-.08407	-.05264	.12054

Q3	-.37458	-.19583	-.00128	.00111	.01416	.21197
Q15	-.37827	-.02638	-.00874	-.10512	.07276	.33999
Q21	-.34485	-.06234	.08184	-.10390	.13425	.07082
Q51	-.70695	-.31158	-.01451	-.05433	-.07339	.19774
Q63	-.78892	-.15129	-.04918	-.12471	-.01822	.10817
Q75	-.49991	-.02421	-.06213	-.09568	.07702	.30147
Q81	-.58063	-.18766	-.06861	.00249	-.00205	.20502
Q86	-.64424	-.13955	-.00829	-.03081	.01063	.20533
Q93	-.78513	-.13681	.01834	-.11178	-.00457	.13481
Q9	.67052	.06061	.09766	.13032	.08277	.05971
Q27	.48499	.42325	.06917	.02868	.06269	-.14705
Q33	.66473	.15442	.02373	.19845	.17029	.04819
Q39	.71133	.24486	.06505	.19146	.13065	.04121
Q45	.50845	.05355	.00368	.22055	.03098	-.03486
Q57	.56530	.35004	.14475	.15697	.13159	.06302
Q69	.64902	.02925	.07369	.23526	.03738	.06331
Q90	.76793	.10399	.02870	.16176	.14729	.04840
Q4	.01363	.17042	.60103	.02590	.02316	-.00709
Q10	.08891	.08048	.62856	.07856	.03634	-.02401
Q16	-.00089	.30492	.63468	-.02867	.04502	.05667
Q22	-.10889	.32931	.61085	-.04845	-.01929	.04628
Q28	.00413	.31917	.75062	.03761	.01876	-.01684
Q34	.06487	.24460	.74003	-.07850	.01221	.06568
Q40	.09283	.18288	.65828	.07794	.14210	.06732
Q46	.10428	.13111	.68149	.06496	.12987	.05774
Q52	.01864	.17668	.79109	-.03360	.03238	.04587
Q58	.02962	.14636	.69580	-.01450	.06571	.17757
Q64	-.02006	.11347	.77219	.02429	-.01348	.09786
Q70	.06841	.12343	.74810	.03331	.03470	.04910
Q76	.00705	-.03394	.42956	.12686	.01826	-.00331
Q82	.01208	.15019	.72204	-.00513	.07545	.18873
Q87	-.00308	.11333	.71667	.08174	.12803	.13011
Q6	-.15080	-.29411	-.01276	-.01837	-.10065	.37794
Q12	-.46585	-.13156	-.04108	-.04227	-.08925	.24124
Q18	-.02507	-.01265	.18930	-.11374	.02260	.47291
Q24	-.46604	-.20972	.06493	-.06876	-.02317	.27548
Q36	-.52172	-.18612	.00426	-.01409	-.02968	.33211
Q54	-.00502	-.16410	-.03814	-.11317	-.19195	.40274
Q60	-.21019	-.16708	.13741	-.01575	-.06066	.44942
Q66	-.01463	.05294	.04762	.01877	.04656	.43354
Q72	-.09683	-.03485	.01493	.06452	-.27855	.22822
Q78	-.09544	-.03832	.11240	-.04587	.09050	.46189
Q84	-.24225	-.15900	06605	-.07460	-.01736	.41369
Q89	-.07209	-.01149	-.07664	-.03908	-.19586	.38570
Q92	-.11925	.02776	.03823	.03295	.05882	.69847
Q95	.02545	-.02861	.15481	-.00063	.14196	.45009
Q97	-.10568	.13544	.23895	-.01313	.07665	.57784
Q103	-.14473	.06840	.09970	-.02034	.07105	.66549
Q30	-.04398	.16343	.04448	.12996	-.04396	-.36631
Q42	.16780	.20023	.02952	.27071	.09884	-.37374
Q48	.13637	.16308	.06278	.27860	.17656	-.39663
Q101	.12978	.05741	.00727	.17790	.16808	-.42126

Table 2
Varimax-rotated Principle Axis Factors
Pattern/Structure Coefficients

	Factor					
Item	I	II	III	IV	V	VI
Q7	.08262	.03388	.08153	.60115	.03685	.07598
Q13	.13686	.06050	.05086	.66842	-.17979	.00072
Q19	.13869	.05224	.02124	.63852	.05701	-.14690
Q25	.12270	.03236	.05932	.64583	.20066	-.11691
Q37	.30214	.06644	.14264	.44535	.10193	-.09726
Q43	.11847	.07934	.01142	.76467	.02699	-.05433
Q49	.15149	.02628	.04815	.65839	-.00517	-.02441
Q55	.12990	-.00058	.01720	.58815	.01045	-.14665
Q61	.09612	.03271	.08705	.67750	-.03386	.01142
Q67	.07197	.00109	.21756	.58143	.02616	-.14650
Q73	.03431	-.01610	.01381	.47157	.04139	-.11139
Q79	.10392	.03080	.04482	.60842	-.02736	.00815
Q85	.10644	.00238	.11789	.63047	.02689	-.06439
Q31	-.16107	-.04431	-.05577	-.58314	.12485	.12827
Q23	-.11619	-.02940	-.07569	-.11941	.37935	.06986
Q29	-.01212	.12720	.01750	.16341	.28389	.07569
Q35	.06773	.05152	.11204	-.04287	.55564	-.01579
Q41	.19482	.13642	.13359	.15330	.39822	.00762
Q47	.06909	-.01119	.03637	-.16026	.57157	-.07526
Q71	.11420	.04218	.08406	.13112	.65221	.03090
Q83	.07299	.03655	.05199	.05054	.70703	.00200
Q88	.08028	.08290	.17085	.31753	.56784	-.15303
Q91	.25423	.31127	.37738	.16019	.38291	.05632
Q94	-.16105	-.02266	-.10323	-.16874	.54555	.13410
Q96	.33219	.09841	.27228	.20525	.26033	-.04065
Q98	-.09327	-.05157	-.01469	-.26016	.40482	.02231
Q100	.12500	.07753	.06595	.09695	.67916	.04215
Q102	.08818	.07845	.17496	.13787	.69850	.01903
Q104	.13295	.06186	-.00173	.07058	.42068	-.06084
Q59	.15851	-.13389	-.11864	.14842	-.42667	-.03562
Q65	-.05876	-.07563	-.10446	.04299	-.41116	-.00697
Q77	.02344	.00504	-.10130	-.12054	-.52555	.11100
Q2	.17034	.22682	.69145	.08998	-.02697	-.08525
Q8	.11711	.19263	.59258	.12380	.09348	-.10371
Q14	.20843	.28916	.71917	.10322	.00943	-.07289
Q20	.20243	.12869	.62051	.18598	.06676	-.06640
Q26	.17058	.23444	.74962	.06857	.12561	-.04322
Q32	.07300	.27216	.49816	.06930	.10898	-.04584
Q38	.21139	.22490	.73769	.12336	.06545	-.03724
Q62	-.21851	-.22226	-.60070	.11090	-.11343	.09300
Q68	.15758	.16355	.64252	.09301	.11130	.00492
Q74	.22959	.25228	.67199	.12268	.17203	-.02204
Q80	.20298	.28245	.66545	.09381	.15345	-.02392
Q44	-.13149	-.23301	-.62370	-.02211	-.05809	.07616
Q50	-.24944	-.10131	-.59135	-.07943	-.08404	.12647
Q56	-.18876	-.09353	-.37954	-.08742	-.06035	.12487

Q3	-.34496	-.00838	-.18687	-.01512	.00417	.21134
Q15	-.34734	.00285	-.04425	-.11211	.06174	.31688
Q21	-.30879	.07089	-.06309	-.10550	.11583	.08813
Q51	-.69563	-.02289	-.30273	-.05977	-.07533	.21308
Q63	-.77946	-.05382	-.14493	-.12807	-.01831	.12361
Q75	-.46201	-.04842	-.04345	-.10506	.06592	.29072
Q81	-.54851	-.07108	-.18431	-.01233	-.00926	.21272
Q86	-.61204	-.00962	-.14286	-.04245	.00642	.21442
Q93	-.77435	.01250	-.13083	-.11534	-.00500	.15094
Q9	.63769	.09394	.06949	.13442	.08050	.04057
Q27	.46812	.08019	.40403	.03992	.06884	-.15571
Q33	.64247	.02701	.15512	.19958	.16629	.03206
Q39	.70034	.06588	.24258	.19368	.13036	.03319
Q45	.46918	.00261	.06767	.21709	.03242	-.05007
Q57	.54850	.15031	.33901	.15993	.13287	.04529
Q69	.61905	.06903	.03946	.23377	.03811	.04754
Q90	.75742	.02915	.10225	.16431	.14570	.03965
Q4	.01659	.56312	.17419	.02857	.02830	.00641
Q10	.08286	.58602	.09483	.07997	.03951	-.01180
Q16	.00615	.61328	.29542	-.02415	.04995	.06329
Q22	-.09513	.59377	.31338	-.04494	-.01205	.05274
Q28	.00741	.74366	.30942	.03958	.02279	-.01443
Q34	.06726	.72774	.23550	-.07419	.01622	.06725
Q40	.09410	.63138	.18344	.07865	.14025	.07477
Q46	.10171	.65383	.13357	.06647	.12977	.06233
Q52	.02046	.78133	.16821	-.03112	.03552	.04789
Q58	.03112	.67261	.14385	-.01189	.06790	.17891
Q64	-.01746	.75210	.11187	.02530	-.01027	.10446
Q70	.06805	.72526	.12237	.03572	.03821	.05473
Q76	.00596	.37638	-.00332	.11718	.01973	.01807
Q82	.01385	.70521	.14450	-.00333	.07689	.18706
Q87	-.00053	.69161	.11451	.08098	.12816	.13740
Q6	-.15780	-.01811	-.27367	-.02824	-.09948	.33952
Q12	-.43191	-.03884	-.13787	-.05506	-.08803	.23670
Q18	-.03376	.18474	-.01500	-.11024	.01916	.42290
Q24	-.43940	.05458	-.20319	-.07933	-.02813	.27336
Q36	-.49550	.00170	-.18452	-.02750	-.03467	.32680
Q54	-.02695	-.03391	-.15525	-.10999	-.17830	.34778
Q60	-.20677	.12718	-.15752	-.02642	-.06056	.41323
Q66	-.02154	.06374	.03675	.01025	.04196	.36496
Q72	-.09213	.01554	-.04103	.04962	-.24892	.19890
Q78	-.09808	.11208	-.03831	-.05098	.07884	.41130
Q84	-.23657	.06230	-.15258	-.08174	-.02169	.37688
Q89	-.07651	-.05594	-.02903	-.04444	-.17671	.32594
Q92	-.10969	.04298	.02161	.02318	.05189	.67488
Q95	.01100	.15025	-.02214	-.00524	.12757	.39711
Q97	-.09712	.23955	.12117	-.01916	.07183	.54291
Q103	-.13386	.10375	.05945	-.02759	.06306	.64010
Q30	-.01872	.03307	.15564	.12284	-.03482	-.30591
Q42	.17519	.02389	.19821	.25931	.09584	-.33092
Q48	.14556	.05189	.16693	.26691	.16687	-.34898
Q101	.13436	-.00491	.06936	.17190	.15574	-.36679